SIMULATION OF THE CURRENT DISTRIBUTION IN A COAXIAL CHANNEL WHEN THE CONDUCTIVITY IS ANISOTROPIC

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We discuss the simulation of the potential and current distributions in a medium with an anisotropic conductivity for the axisymmetric case.

Solutions of the problem of the potential and current distributions in a medium having an anisotropic conductivity have appeared in a number of papers [1-3]. The solution of three-dimensional problems when the Hall constant $\omega\tau \neq 0$ is very difficult. In some special cases the solution is simplified since electric simulation is possible by using a material with an appropriate conductivity tensor [4]. Simulating an anisotropic conductivity presents certain technical difficulties, however [4, 5].

Methods are described for producing models with an anisotropic conductivity for the two-dimensional case only.

A simple solution of the problem in the axisymmetric case (r, θ, z axes) is possible under the following simplifying assumptions:

1) the electrical conductivity of the medium σ = const;

2) the Hall constant $\omega\tau$ = const;

3) the magnetic induction B has components (0, 0, B_Z);

4) the electric field can be described by a potential φ;

5) the velocity V has components (0, 0, V_Z).

The potential and current distribution patterns can be found by solving the equations

$$\operatorname{div} \mathbf{j} = 0, \quad \sigma \operatorname{grad} \varphi = \mathbf{j} + \frac{\omega\tau}{|B|} \mathbf{j} \times \mathbf{B} \tag{1}$$

where j is the current density, with the boundary conditions on a conductor

$$\varphi = \text{const}, \quad j_\tau = \frac{\omega^2\tau^2 B_n B_\tau / B^2}{1 + \omega^2\tau^2 (B_n / B)^2} j_n \tag{2}$$

and on an insulator

$$\frac{\partial\varphi}{\partial n} = -\frac{\omega^2\tau^2 B_n B_\tau / B^2}{1 + \omega^2\tau^2 (B_n / B)^2} \frac{\partial\varphi}{\partial\tau} \tag{3}$$

In Eqs. (2) and (3) n and τ are the outward normal and tangent to the channel wall. To the boundary conditions (2) and (3) are added the asymptotic conditions at infinity and the condition that no current flows toward the axis of symmetry.

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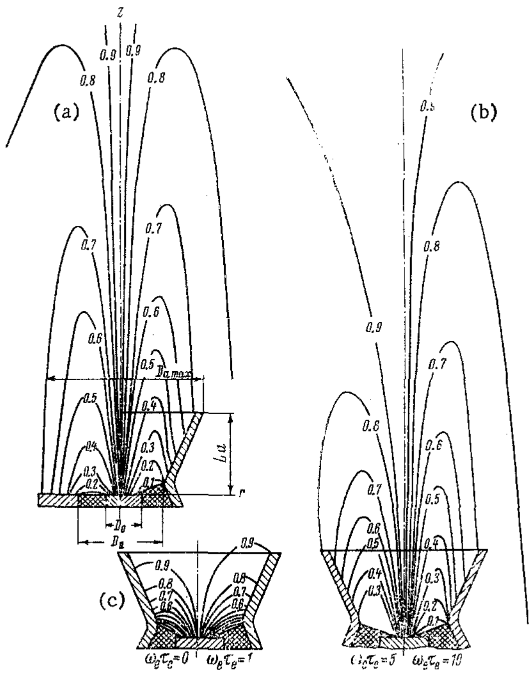

Fig. 1

It is clear that boundary conditions (2) and (3) are difficult to simulate. At the same time it is easy to show that replacing the independent variables r and z by r and z', where $z' = z/\sqrt{1+\omega^2\tau^2}$, reduces the problem under discussion to that of solving Laplace's equation

$$\frac{\partial^2\varphi}{\partial z'^2} + \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial\varphi}{\partial r} = 0 \tag{4}$$

and the corresponding equation [6] for the current function U

$$\frac{\partial}{\partial z'}\frac{1}{r}\frac{\partial U}{\partial z'} + \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial U}{\partial r} = 0, \quad \frac{\partial U}{\partial r} = 2\pi r j_z, \quad \frac{\partial U}{\partial z'} = -2\pi r j_r \tag{5}$$

with boundary conditions at infinity and on the axis of symmetry, and also with the conditions φ = const, $j_\tau = 0$ on a conductor and $j_n = 0$ on an insulator.

The solution of (4) with the boundary conditions stated can be obtained by simulation in an electrolytic tank [6]. The thickness δ of the layer of electrolyte in the tank must satisfy the relation $\delta(r, z') = kr$, where k is a constant.

Returning to the problem of the nonuniform medium Eq. (5) can be solved in an electrolytic tank. In this case the thickness of the layer of electrolyte in the tank must satisfy the relation $\delta(r, z') = k/r$, where k is a constant. A copper plate is placed along the axis of symmetry to limit δ at $r = 0$.

The structure of the channel studied is shown in Fig. 1a. A current of intermediate frequency was used in the simulation. A bridge circuit was used to find the potential distribution in the electrolyte.

In accord with the change of independent variables introduced, the magnitude of $\omega\tau$ determines the deformation of the channel necessary for its simulation in r, z' coordinates. In making the model of the channel in r, z' coordinates the scale in the z' direction is chosen $\sqrt{1+\omega^2\tau^2}$ times smaller than in the r direction. For sufficiently large values of $\omega\tau$ such that $D_{a\,max} \gg L_a/\sqrt{1+\omega^2\tau^2}$ and any shape channel, the configuration of electrodes in the rz' plane will be only slightly different from electrodes in the form of thin disks of diameters D_0 and $D_{a\,max} - D_a$ and placed in the plane $z' = 0$ (Fig. 1a). The quantities D_0, D_a, $D_{a\,max}$, and L_a are shown in Fig. 1a.

To solve system (1) the distribution obtained in the simulation in rz' coordinates must be transformed to rz.

It follows from the above that for $\omega\tau \gg L_a/D_{a\,max}$ the shape of the electrodes does not significantly affect the potential distribution or the currents outside the outer electrode. The results shown in Fig. 1a illustrate this assumption. Figure 1a shows a graph of the current distribution for $\omega\tau = 10$ for two different electrode configurations. The part of the graph to the right of the axis of symmetry corresponds to an experiment in which the outer electrode was conical as shown in the figure. In obtaining the measurements shown on the left side of the graph the outer electrode was a disk. It is clear that for $z \gg L_a$ the current distributions in the two experiments are only slightly different from one another.

Some of the current distributions obtained in the simulation of the channel in the rz plane are shown in Fig. 1b and c. These figures show the effect of $\omega\tau$ on the current distribution. The current distribution in Fig. 1c to the left of the axis of symmetry is for $\omega\tau = 0$ and that to the right for $\omega\tau = 1$. In Fig. 1b $\omega\tau = 5$ to the left of the axis of symmetry and $\omega\tau = 10$ to the right. It is clear that the region occupied by the currents increases with $\omega\tau$. For large $\omega\tau$ the region occupied by the currents in the direction of the z axis increases linearly with $\omega\tau$ since $z \approx z'\omega\tau$.

The azimuthal Hall current j_0 can be calculated from the known current distribution in the rz plane. It follows from Eq. (1) that

$$j_\theta = \omega\tau j_r = -\frac{\omega\tau}{\sqrt{1+\omega^2\tau^2}}\frac{1}{2\pi r}\frac{\partial U(r, z')}{\partial z'}$$

When $\omega\tau$ is large, the function $U(r, z')$ and its derivative $\partial U/\partial z'$ vary slowly with the Hall constant so that the azimuthal current density distribution in rz' coordinates will also vary slowly with $\omega\tau$. The azimuthal current density hardly changes as $\omega\tau$ increases, and the total azimuthal current

$$U_\theta = \int_S j_\theta dr dz$$

will increase linearly with $\omega\tau$ since $z \approx z'\omega\tau$.

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